Parameter-Synthesis Problems for One-Counter Automata

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(slides by Ritam Raha)

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Universiteit Antwerpen

Outline

1. One-Counter Automata (Parametric) and Synthesis Problem
2. Previous Approach
3. Approach with Alternating Two-Way Automata for a subclass
4. (Failed) Approach with Partial Observation Games

One-Counter Automata

```
n = 5
n = max(0, n - 5)
if n = 0:
  while n < 10:
    n = n + 1
  n = n + 100
  # make progress
else:
  assert(False)
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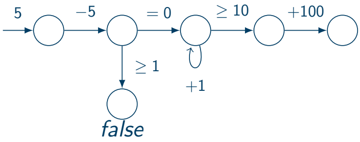

- Configurations: $(q, c), c \geq 0$;

One-Counter Automata

```python
def foobar(x):
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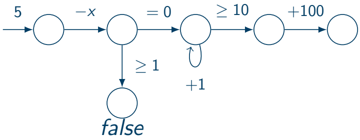
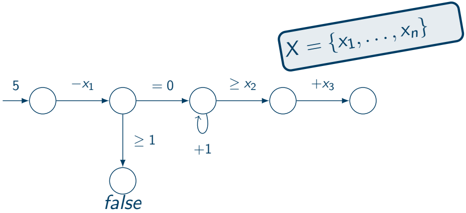


Parametric One-Counter Automata



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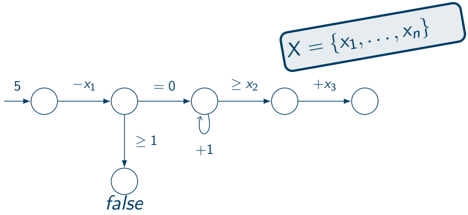

Definition (Succinct OCA with Parameters)

$\mathcal{A} = (Q, q_{in}, T, \delta, X)$

$\delta : T \to Op$ with Op the union of

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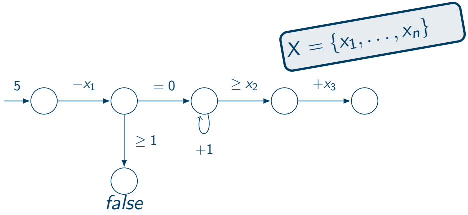

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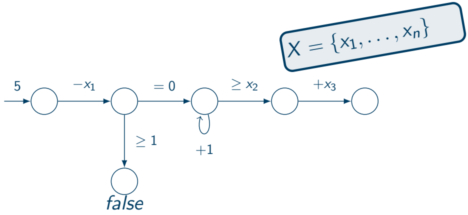

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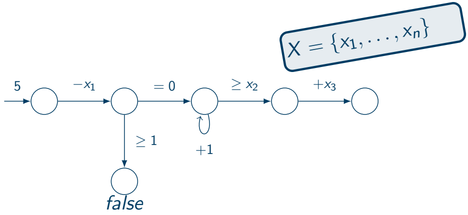

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Non-parametric: $X = \emptyset$

One-Counter Automata

Models		*CU*	*PU*	*ZT*	*PT*
Non-Parametric	SOCA	✓	✗	✓	✗
Parametric	OCAPT	$\{-1, 0, 1\}$	✗	✓	✓
	SOCAP	✓	✓	✓	✓

Decision Problems

Non-Parametric:

Parametric:

Decision Problems

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Reach

$\exists \rho$ such that $(q_{in}, 0) \xrightarrow{\rho} q_f$

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[NP-complete (HKOW'09)]

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$\exists V : X \to \mathbb{N}$ s.t. $\exists \rho$, $(q_{in}, 0) \xrightarrow{\rho}_V q_f$

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[in NEXP (HKOW'09,LOW'15)]

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$\exists V : X \to \mathbb{N}$ s.t. $\exists \rho$,
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UnivReach

For all infinite ρ,
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UnivReach for SOCA

Model and problem

- $Op = CU \cup ZT$
- UNIVREACH: all infinite paths reach q_f

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$\exists$ infinite path avoiding q_f = two REACH queries (coNP)

Hardness from reduction from co-SUBSETSUM.

Proposition

The UNIVREACH *problem for SOCA is coNP-complete.*

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Full PAD is undecidable; one alternation suffices for undecidability.

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Theorem (Lipshitz'78, Lechner-Ouaknine-Worrell'15)

The existential fragment of PAD (EPAD) is decidable in NEXP.

$\forall\exists_R$PAD & Undecidability

- $\forall\exists_R\text{PAD} := \forall z_1 \ldots \forall z_n \exists x_1 \ldots \exists x_m . \varphi(\mathbf{x}, \mathbf{z})$
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$\neg(a \mid b) \equiv b = aq + r$ where, $0 < r < b$.

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3. Multiplication

Undecidable!!

Decision Problems

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Outline

Alternating Two-Way Automaton

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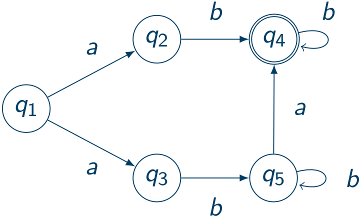

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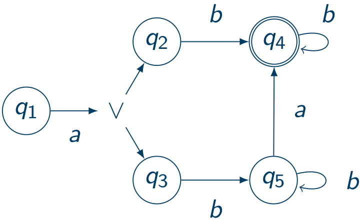

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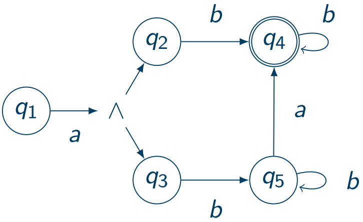

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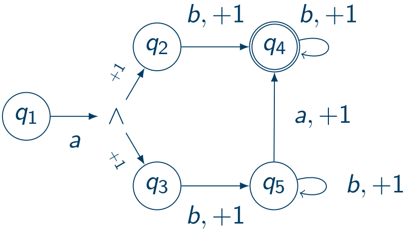

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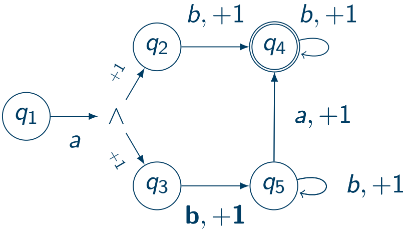

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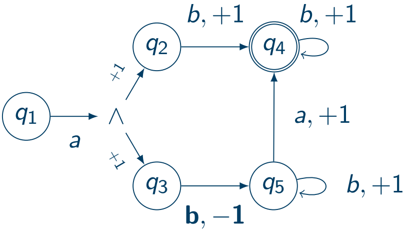

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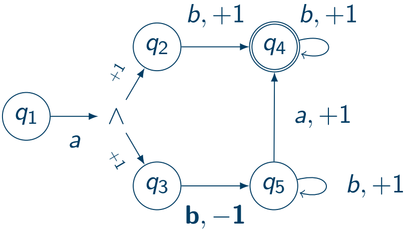

Theorem (Serre'06)

The non-emptiness problem for A2As is in PSPACE.

From OCAPT to A2A

$Op = \{-1, 0, +1\} \cup ZT \cup PT$

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Proposition (Based on Bollig-Quaas-Sangnier'19)

For every OCAPT $\mathcal{A}$ we construct an A2A $\mathcal{T}$ of poly-size which accepts words corresponding to valuations that witness SynthReach *is true.*

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Idea:

- Encode valuations as parameter words
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- Accept the reaching runs
- (...and runs that "die off")

OCAPT to A2A

Valuation to words:

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$V : X \to \mathbb{N};\qquad \Sigma = X \cup \{\square\};$

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$\square$

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$\square\square\square x_1 \square x_2 \square^\omega$

Valuation to words:

$V : X \to \mathbb{N}$; $\Sigma = X \cup \{\Box\}$;

$\Box\Box\Box x_1 \Box x_2 \Box^\omega \Rightarrow x_1 \mapsto 2, x_2 \mapsto 3$;

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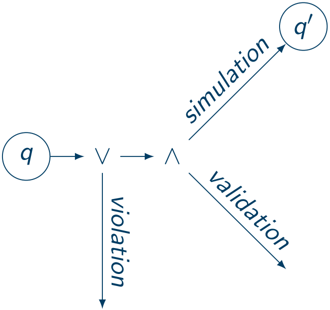

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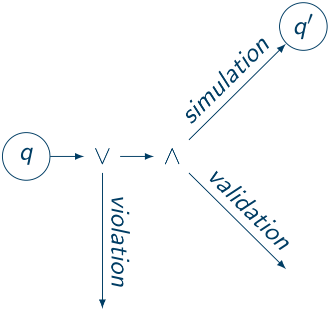

Accept reaching runs:

OCAPT to A2A

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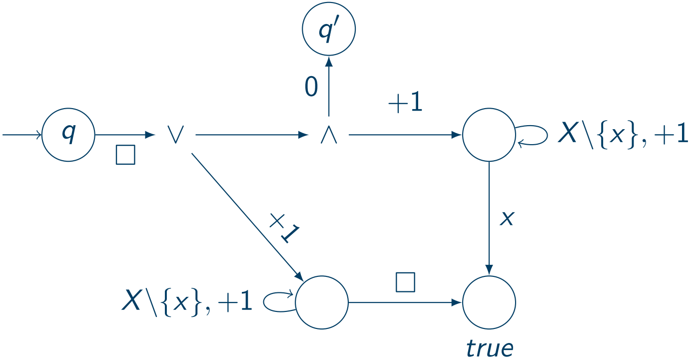

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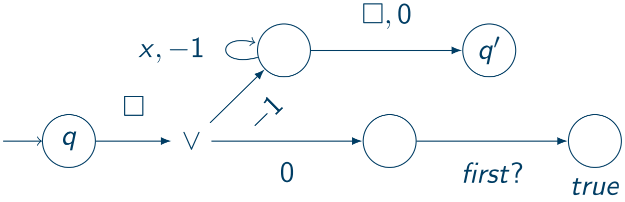

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OCAPT to A2A

We want to check all runs:

$$q \xrightarrow{op_1} q_1 \Rightarrow (q, \Box, \mathcal{T}_1)$$

$$q \xrightarrow{op_2} q_2 \Rightarrow (q, \Box, \mathcal{T}_2)$$

$$(q, \Box, \mathcal{T}_1 \wedge \mathcal{T}_2) \in \mathcal{T}$$

OCAPT to A2A

Proposition

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Proposition

SynthReach *for OCAPT is in PSPACE.*

- For every A2A $\mathcal{T}$ there is an NBA $\mathcal{B}$ of exponential size accepting same language
- Non-emptiness witnesses for NBAs are simple "lassos"
- $\implies$ SynthReach admits exponential (w.r.t. the OCAPT) witnesses and thus polynomial in binary encoding

OCAPT to A2A

Proposition

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- For every A2A $\mathcal{T}$ there is an NBA $\mathcal{B}$ of exponential size accepting same language
- Non-emptiness witnesses for NBAs are simple "lassos"
- $\implies$ SYNTHREACH admits exponential (w.r.t. the OCAPT) witnesses and thus polynomial in binary encoding
- Guess a valuation and check UNIVREACH for resulting SOCA

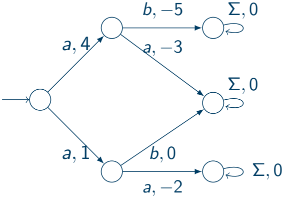
Theorem

SYNTHREACH *for OCAPT is in* NP^{coNP}.

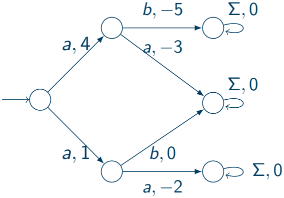
Outline

1. One-Counter Automata (Parametric) and Synthesis Problem
2. Previous Approach
3. Approach with Alternating Two-Way Automata for a subclass
4. Approach with Partial Observation Games

Partial Observation Energy Games



Partial Observation Energy Games



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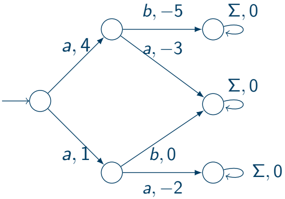
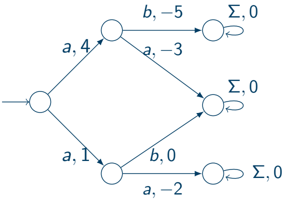


- Chooses an action

- Resolves non-determinism

Partial Observation Energy Games



- Chooses an action
- Keeps the energy level positive

- Resolves non-determinism
- Wants it eventually negative

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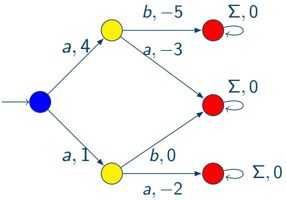

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Conjecture

SynthReach *problem for SOCAP is decidable.*

Decision Problems

Non-Parametric:

REACH

$\exists \rho$ such that $(q_{in}, 0) \xrightarrow{\rho} q_f$

[NP-complete]

UNIVREACH

For all infinite ρ, $(q_{in}, 0) \xrightarrow{\rho} q_f$

[coNP-complete]

Parametric:

PAR-REACH

$\exists V : X \to \mathbb{N}$ s.t. $\exists \rho$, $(q_{in}, 0) \xrightarrow{\rho}_V q_f$

[in NEXP]

SYNTHREACH

$\exists V$ s.t. for all infinite ρ, $(q_{in}, 0) \xrightarrow{\rho}_V q_f$

[Decidable?]

Conclusion

- If parameters are only allowed on tests, the problem is in NP^{NP}
- In full generality, for SOCAP it is still open